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TOWARD THE PHENOMENOLOGICAL THEORY OF DRYING

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Interrelated processes of the transfer of heat, mass, and deformation of disperse systems with a prevailing role of coagulation links in the structure are considered. The determining processes occurring during their drying under ordinary conditions are singled out. The procedure for calculating large displacements and deformations on shrinkage of material during its drying has been developed. Basic rheological bodies are used in calculations. The possibility of destruction of an elastic material is considered.

Introduction. When being dried, almost all materials are prone to deformation and cracking [1]. These are all kinds of farm products, foodstuffs, building materials, rocks, textile fabrics, etc. However, as yet the study of deformation and cracking in the process of drying has been inadequate. Despite the fact that the necessity of accounting for deformation in the process of drying was understood as soon as drying began to be used, its practical implementation is connected with great difficulties of both experimental and theoretical nature. In experiments one faces problems associated with the technical feasibility of measuring deformations and stresses, the accuracy with which physical quantities are measured, and with the reproduction of experimental data. Theoretical description entails allowance for interrelated nonlinear physical phenomena, the necessity of solving complex systems of differential equations, and the development of new numerical methods of calculation.

Drying is the interrelated process of heat and mass transfer and of the developing strain-stressed state of the material. While the processes of heat- and mass transfer have been well studied (a vast literature is available, see [1–8]), shrinkage and related stresses in capillary-porous colloid systems have been considered inadequately, and much less attention is paid in scientific literature to this aspect. Therefore, the aim of the present investigation is the statement of the problem of heat- and mass transfer with allowance for the deformation of the material and the stresses appearing in it and the development of a method for its solution. Emphasis is put on the study of the processes of material deformation as a constituent of drying on par with the processes of heat- and mass transfer.

General Ideas. In materials being dried, a great role is played by coagulations which are solid-phase particles linked via thin water interlayers [9–12] which allow materials to deform strongly and change their shape without loss of integrity in case they are subjected to external loading. Water in the material is a source of capillary forces and disjoining pressures [2, 13, 14] which may attain high values in the process of its drying and lead to shrinkage, warping, cracking, and even destruction.

In the course of deformation of disperse systems, their structure undergoes a change caused by rearrangement of particles, breakage of the former structural links between the latter, and formation of new ones. Precisely this capability of bodies is responsible for many of their rheological properties, including yielding. Thus, under insignificant loadings on a material one can distinguish instantaneous elastic and slow deformations mainly proceeding with a change in the volume of the pores and without breakage of the structure. A further increase in loading leads to a further deformation of the skeleton structure and to breakage of interparticle links where they are most weak. Simultaneously, new links are being formed at other points of interparticle contacts. This is accompanied by the establishment of a dynamic equilibrium between the breaking and recovering links. A further increase in loading upsets this equilibrium, i.e., the number of broken links starts to exceed those recovered. This causes the appearance of defects in a body and an increase in loading on the remaining links. Subsequent increase in loading leads to the formation and growth of cracks in the material and to its destruction.

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Theoretically, the deformation of a material and the change in its structure can be caused by gradients of temperature, moisture content, or of total pressure. The interrelationship between the processes of transfer and structure transformation is attributable to general causes: the dispersity of particles and the field of surface forces [2]. In the course of heat- and mass transfer in disperse systems subjected to deformation there occurs a change in the mutual position of particles, number of contacts between them, average dimensions of pores, and in the curves of pore-radius distribution, which makes the rate and mechanism of heat- and mass transfer processes dependent also on structure rearrangement. Consequently, there also occurs a change in the coefficients of transfer which become functions of both temperature and moisture content, as well as of the rearranging structure of the disperse system.

Attempts have been made to justify and derive general equations of interrelated processes of heat- and mass transfer and of deformation on the basis of the thermodynamics of irreversible processes [3, 15–21], in which it is assumed that the fluxes of mass and heat and the rate of change in the filtrational pressure and deformation depend on the gradients of moisture content, temperature, and total pressure in the material, as well as on the rate of its deformation. Specific methods of derivation of these equations are cited in the references given. However, this single theory cannot take into account the entirety of the occurring processes. Thus, the interrelationship of the above-indicated fluxes is reflected not only via the corresponding gradients of fields, but also via the dependence of all the physical coefficients (coefficients of heat conduction and diffusion, heat capacity, etc.) on temperature, moisture content, material deformation, and, in particular, on the rearranging structure of a disperse system.

The practice of studying real, simultaneously occurring processes shows that all of the fluxes listed above are never of the same order of magnitude. Depending on specific conditions, one can always single out a dominating phenomenon which is more significant than the remaining ones.

The mechanics of continua is based on the equations of continuity, motion, and moment of momentum. In order to close the system of equations that describes the behavior of a specific continuous medium, it is necessary to know the rheological equation of the latter [22–28]. In the case of appreciable shrinkages and deformations of the material it is already insufficient to use Hooke's law, whereas for the systems we are considering the viscous and plastic properties of the material can manifest themselves simultaneously with the elastic ones from the very beginning of the process of loading. The complex character of the structures of disperse systems is responsible for their various and complex rheological properties. However, it is known that the rheology of the material can be described on the basis of three ideal bodies — the Hooke body, the Newton body, and the Saint Venant body — corresponding to which are three fundamental properties: elasticity, viscosity, and plasticity. The rest of the rheological bodies are considered as combinations of these three basic ones, and their properties are correspondingly a combination of the fundamental ones. Therefore, real systems can be modeled with the aid of different combinations of ideal bodies. Hence it is logical to study the processes of heat- and mass transfer and deformation for the materials that possesses elasticity or viscoelasticity, or elastoplasticity. They can be used as a basis for investigating the processes of heat- and mass transfer and deformation of materials of a more complex rheological behavior.

For disperse systems with a predominant role of coagulation links great shrinkages are typical. In contrast, for example, to the thermal elasticity of metals, where thermal deformations of material are small and do not exceed a percent of the initial size of a body, drying of bodies with coagulation structures can cause changes in size which can amount not to percentages but rather to severalfold. The description of large displacements of elements and deformation of a body requires the development of new computing methods. Application of the Euler method to study mechanical motion is unjustifiable; one has to use the Lagrange method and take into account the geometrical nonlinearity of the problem. Consequently, it is impossible in principle to solve such problems analytically. In our opinion, the reduction of the problem to a geometrically linear one, i.e., to small deformations in a system, is transition to another problem rather than the simplification of the one considered. Moreover, the problem is nonlinear also by virtue of the dependence of physical characteristics on the moisture content, temperature, and deformations of material. Its solution is complicated by the necessity of taking into account the interrelationship between the unknown quantities. However, due to the development of numerical methods and computational technique, it becomes possible to solve nonlinear interrelated problems of drying that account for large deformations of material.

Thus, the aim of the present investigation is the singling out of the main specific features of the problem considered and development of the technique of its solution, and thereby the generalization of the theory of drying to the case of large displacements and deformations in a material.

Statement of the Problem. As noted above, drying consists of interrelated processes of transfer of heat, mass, and of total pressure in the material and of its deformation. However, because of the absence of a flux of moisture, drying becomes impossible, whereas it can take place also at constant temperature and at constant pressure or at their small gradients, so that mass transfer due to them can be neglected. Consequently, the first process which should be taken into account is moisture transfer. In the overwhelming majority of disperse systems with coagulation links shrinkage occurs if the systems are left to their own devices or there appear stresses if their shape and volume cannot be changed. Therefore, the second process which should be considered is deformation of material and the associated stressed state. All the remaining processes can be present depending on the specific conditions of drying.

The study of drying accompanied by deformation has shown that the mutual influence of heat- and mass transfer and of deformed-stressed state of material manifests itself only in the case of a very unsteady heat- and mass exchange with the environment. Only in such a case should one take into account inertial forces. In ordinary processes of drying, the fields of temperature, moisture content, and of total pressure change slowly, and the material has time to respond to their changes, so that with satisfactory accuracy one can assume that the body is in static equilibrium all the time. In ordinary drying of disperse systems their deformed-stressed states weakly influence changes in temperature, moisture content, and in total pressure. This is also confirmed by the calculations made in [29]. Therefore, in the equations of heat conduction, mass transfer, and of a change in total pressure one can neglect the terms that describe the deformed-stressed state of material, whereas in equations of mechanical motion one can omit inertia terms. Moreover, thermal deformations and the deformations caused by the total pressure in the material can be neglected in comparison with the shrinkages caused by dehydration. We clarify that by ordinary drying we understand convective drying at a temperature below 100^{o}C, relative air humidity lower than 100%, and velocity of motion of the drying agent of up to 7 m/sec.

With allowance for the basic effects studied and without loss of generality we may consider the following statement of the problem. Mass transfer occurs under isothermal conditions. On the upper boundary Newton-law mass transfer is prescribed and on the remaining ones moisture insulation from the environment is assumed. We shall restrict ourselves to a simple uncoupled moisture-stressed problem in which the influence of mechanical motion on mass transfer is not taken into account. In the process of drying, at each time instant a body is in mechanical equilibrium, which allows one to investigate a static problem of moisture elasticity. We will also consider a two-dimensional problem under the conditions of a plane stressed state. We assume that the rheological and mass-transfer coefficients do not depend on the moisture content of the material. In the present work, we will study an elastic, an elastoviscous, and an elastoplastic bodies, as well as an elastic body with a possibility of cracking.

We note that allowance for the dependence of the above-mentioned coefficients on the sought-for quantities can be made within the framework of the procedure we are developing, since the solutions of such nonlinear problems have been well described. In our opinion, the generalization of the problem to the case of interrelated heat- and mass transfer involves no fundamental difficulties, since the equations of heat conduction and of total pressure have the same structure as the mass conduction equation.

Thus, the model considered takes into account all the necessary properties and at the same time is so simple that it allows one to develop the fundamental notions of the computing method suggested.

Solution Procedure. The equations of mechanical motion of an elastic material in Lagrangian curvilinear coordinates in a linear approximation are given in [30]. It is seen that they are cumbersome and are hardly solvable analytically. With allowance for the nonlinear components in the expressions for the tensor of deformations new terms are added to those already available in the equations of motion. Due to this, the system studied becomes more complicated and less comprehensible. The solution of such equations by the grid method is possible, but it is very laborious because of the necessity to take into account the deformability of the computational grid and the change in the shape of the material boundaries, and the development of the techniques that allow one to preserve the needed accuracy of calculation.

In the present work, to determine large displacements and deformations a finite-element method is used. We will write variational formulations for the processes of motion and mass transfer as follows [31, 32]:

$$\delta \int_V \frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij} - \sigma_{ij}\varepsilon_{0ij}\right) dV = 0 \,, \qquad (1)$$

$$\delta\left\{\int\limits_V\left[a_w\left(\frac{\partial W}{\partial x}\right)^2+a_w\left(\frac{\partial W}{\partial y}\right)^2-2W\frac{\partial W}{\partial t}\right]dV+\int\limits_S \alpha_w\,(W-W_{eq})^2\,dS\right\}=0\,. \tag{2}$$

The general procedure of their solution is given in [30]. Its main stages are: by selecting a time step Δt small enough that the moisture-gradient caused displacements can be considered insignificant, we obtain the distribution W from Eq. (2) and then from Eq. (1) we find the displacements of nodal points which correspond to equilibrium for a new distribution of moisture. On the basis of the displacements obtained we calculate new values of the coordinates of the nodes of elements, and this gives a new equilibrium deformed state of the material. Taking the resulting state of the material as the initial one, we repeat the calculation on the next step.

The system of algebraic equations was constructed on the basis of the procedure presented in [31, 32]. The general scheme of solution is based on finding the displacements of the nodes of elements on each step. For this purpose, it is necessary to use the dependence $\{\sigma\} = f(\{\varepsilon\})$ in Eq. (1) for various rheological bodies to exclude σ_{ij}.

The rheology of the material which obeys Hooke's law will be expressed by the well-known equation

$$\sigma_{ij}=\frac{\nu E}{(1+\nu)\,(1-2\nu)}\,\varepsilon_{kk}\delta_{ij}+\frac{E}{1+\nu}\,\varepsilon_{ij}\,. \tag{3}$$

Many viscous properties of disperse systems are satisfactorily described by the model of a standard linear elastoviscous body which consists of successively connected Hookean and Kelvin bodies [25, 33, 34]. In [35], a general equation for the dependence of the stress tensor on the deformation tensor is obtained when a material dried is under the conditions of a complex stressed state, as for example, in the case of drying:

$$\sigma_{ij}\,(t)=2G_0\left(\varepsilon_{ij}\,(t)-\frac{1}{3}\,\delta_{ij}\theta\,(t)\right)+\delta_{ij}B\theta\,(t)-2G_0\int\limits_0^t R_s\,(t-\tau)\left(\varepsilon_{ij}\,(\tau)-\frac{1}{3}\,\delta_{ij}\theta\,(\tau)\right)d\tau\,. \tag{4}$$

Here, $R_s(t-\tau) = \dfrac{G_0-G_\infty}{G_0t_r}\,\exp\left[-\dfrac{t-\tau}{t_r}\right]$, $\theta = \varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}$.

To describe the plasticity of the material, the theory of Prandtl–Reiss plastic flow and Mises yielding condition [36, 37] were used:

$$de_{ij}=s_{ij}d\lambda+\frac{ds_{ij}}{2G}\,,\quad d\lambda=\frac{3}{2}\frac{\overline{d\varepsilon^{pl}}}{\bar\sigma}=\frac{3}{2}\frac{d\bar\sigma}{\bar\sigma H}\,,\quad \overline{d\varepsilon^{pl}}=\sqrt{\frac{3}{2}\,d\varepsilon_{ij}^{pl}\,d\varepsilon_{ij}^{pl}}\,,\quad H=\frac{d\bar\sigma}{\overline{d\varepsilon^{pl}}}\,, \tag{5}$$

$$\bar\sigma=\sqrt{\frac{3}{2}\,s_{ij}s_{ij}}\,. \tag{6}$$

As is known [36, 37], for a plastic material one cannot write finite relations between the stress and deformation tensors; however, one can obtain relationship between their differentials. Their derivation on the basis of Eqs. (5) and (6) is presented in [38]. We will present here only the finite relationship between the differentials of the stress and strain tensors:

$$d\sigma_{ij}=\frac{E}{2\,(1+\nu)}\left(d\varepsilon_{ij}+\frac{\nu}{1-2\nu}\,\delta_{ij}d\varepsilon_{ij}-s_{ij}\,\frac{s_{kl}d\varepsilon_{kl}}{S}\right),\quad S=\frac{2}{3}\,\bar\sigma^{\,2}\left(1+\frac{2\,(1+\nu)\,H}{3E}\right). \tag{7}$$

Now, we will present relationships between the stress and strain vectors $\{\sigma\} = f(\{\varepsilon\})$. For an elastic material, in the case of a plane stressed state with account for the dependence of the stress tensor on the moisture content of the material we have

$$\{\sigma\} = [D]\,\{\varepsilon\} - [D]\,\{\varepsilon_0\}\,, \tag{8}$$

$$[D] = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{1+\nu} \end{bmatrix}. \tag{9}$$

Here, $\varepsilon_{0ij} = \beta_{\text{v}}(W - W_0)\delta_{ij}$ is the deformation caused by the difference of moisture contents; W_0 is the moisture content at the initial instant of time t_0 (the initial moisture content of the material).

For a viscoelastic material, in the case of a plane stressed state, we may write the following equation which relates the stress and strain tensors and which was derived in [35]:

$$\{\sigma\} = [D]\,\{\varepsilon\} - [D]\,\{\varepsilon_0\} + \{r\}\,. \tag{10}$$

Here, the matrix $[D]$ is given by Eq. (9), and the components of the vector $\{r\}$ have the form

$$r_1 = -\int_0^t R_{\text{s}}\,(t-\tau)\left[\frac{(2-\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{11}\,(\tau) - \frac{(1-2\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{22}\,(\tau)\right]d\tau\,, \tag{11}$$

$$r_2 = -\int_0^t R_{\text{s}}\,(t-\tau)\left[\frac{(2-\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{22}\,(\tau) - \frac{(2-\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{11}\,(\tau)\right]d\tau\,, \tag{12}$$

$$r_3 = -\frac{E}{2\,(1+\nu)}\int_0^t R_{\text{s}}\,(t-\tau)\,\varepsilon_{21}\,(\tau)\,d\tau\,. \tag{13}$$

For an unstrengthened elastoplastic body in the case of a plane stressed state on the basis of the results of [38] the equation for the differential of the stress tensor has a form similar to that of relation (8) for an elastic body, in which the quantities σ and ε should be replaced by their differentials $d\sigma$ and $d\varepsilon$. However, for plastic elements the matrix $[D]$ should be replaced by the matrix $[D^{\text{pl}}]$ which follows from Eq. (7) and which was derived in [38]:

$$\left[D^{\text{pl}}\right] = \varphi\,\frac{E}{Q}\begin{bmatrix} s_{yy}^2 + 2P & -s_{xx}s_{yy} + 2\nu P & -\frac{s_{xx}+\nu s_{yy}}{1+\nu}\,\sigma_{xy} \\ -s_{xx}s_{yy} + 2\nu P & s_{yy}^2 + 2P & -\frac{s_{yy}+\nu s_{xx}}{1+\nu}\,s_{xy} \\ -\frac{s_{xx}+\nu s_{yy}}{1+\nu}\,s_{xy} & -\frac{s_{yy}+\nu s_{xx}}{1+\nu}\,s_{xy} & \frac{R}{2\,(1+\nu)} \end{bmatrix}, \tag{14}$$

where $P = \sigma_{xy}^2/(1+\nu)$; $R = s_{xx}^2 + 2\nu s_{xx}s_{yy} + s_{yy}^2$; $Q = R + 2(1-\nu^2)P$; φ is a specific quantity assumed to be equal to unity for a plastic material.

We have also considered the model of an elastically destroyed material which was constructed on the basis of Eq. (8) using the matrices $[D]$ and $[D^{\text{pl}}]$, but sinse the elements on which destruction occurred cannot influence the entire process of deformation, the quantity φ in Eq. (14) was assumed rather small (in our case, it was equal to 0.001).

The body investigated was split into two-dimensional simplex-elements, which made it possible to use linear interpolation polynomials in solving the problem. The components of the vector of displacements through their nodal values $\{U\}$ will be written as follows:

$$\{u\} = [N]\,\{U\}\,. \tag{15}$$

The strain vector $\{\varepsilon\}$ is related to nodal displacements $\{U\}$ by the expression

$$\{\varepsilon\} = [B]\,\{U\}\,. \tag{16}$$

For an elastic material the substitution of Eq. (8) into Eq. (1) allows us to exclude the stress tensor form the variational formulation. Then, by applying Eq. (16) to the expression obtained and performing differentiation with respect to $\{U\}$ and thereafter integration over V, we finally obtain a system of algebraic equations of the form [30]

$$[K]\,\{U\} = \{F\} - [K]\,\{U_0\}\,. \tag{17}$$

Here,

$$[K] = \sum_{e=1}^{N} \left[K^e\right]; \tag{18}$$

$$\{F\} = \sum_{e=1}^{N} \left\{F^e\right\}; \tag{19}$$

$$\left[K^e\right] = \left[B^e\right]^{\mathrm{t}} \left[D^e\right]\left[B^e\right] S^e h\,; \tag{20}$$

$$\left\{F^e\right\} = \left[B^e\right]^{\mathrm{t}} \left[D^e\right]\left\{\varepsilon_0^e\right\} S^e h\,. \tag{21}$$

Using Eq. (10) and performing the same operations as for the elastic body, we obtain a system of algebraic equations for an elastoviscous material [35]:

$$[K]\,\{U\} = \{F\} - [K]\,\{U_0\} + \{R\}\,. \tag{22}$$

The meaning of the first two terms on the right is the same as before, whereas the third term takes into account the hereditary properties of the material and has the form

$$\{R\} = \sum_{e=1}^{N} \left\{R^e\right\}, \tag{23}$$

$$\left\{R^e\right\} = \frac{1}{2}\left[B^e\right]^{\mathrm{t}} \left\{\rho^e\right\} S^e h\,. \tag{24}$$

Here, the components of the vector $\{\rho\}$ can be found from expressions (11)–(13) for the components of the vector $\{r\}$:

$$\rho_1 = \sum_{i=1}^{M-1} R_{\mathrm{s}}\,(t-\tau_i)\left[\frac{(2-\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{11}\,(\tau_i) - \frac{(1-2\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{22}\,(\tau_i)\right]\Delta\tau_i\,, \tag{25}$$

$$\rho_2 = \sum_{i=1}^{M-1} R_s\,(t-\tau_i)\left[\frac{(2-\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{22}\,(\tau_i) - \frac{(1-2\nu)\,E}{3\,(1-\nu^2)}\,\varepsilon_{11}\,(\tau_i)\right]\Delta\tau_i\,, \tag{26}$$

$$\rho_3 = \frac{(2-\nu)\,E}{3\,(1-\nu^2)}\sum_{i=1}^{M-1} R_s\,(t-\tau_i)\,\varepsilon_{21}\,(\tau_i)\,\Delta\tau_i\,. \tag{27}$$

We note that $\{\rho\}$ is calculated each time at a new step, since the number of steps in time M increases as the calculation proceeds. It is recalculated, first, because new terms are added into the sums of Eqs. (25)–(27) and additional new quantities τ_i, $\varepsilon_{11}(\tau_i)$, $\varepsilon_{22}(\tau_i)$, and $\varepsilon_{21}(\tau_i)$ are introduced and, second, because the function $R_s(t-\tau_i)$ depends on t, and therefore, it should be calculated anew at each step.

For an elastoplastic material the system of algebraic equations will have the following form suggested by the author in [39]:

$$[K]\,\{U\} = \{F\} - [K]\,\{U_0\} + \{F^{\,\mathrm{pl}}\}\,, \tag{28}$$

$$[K^{e\mathrm{pl}}] = [B^e]^{\,\mathrm{t}}\,[D^{e\mathrm{pl}}]\,[B^e]\,S^e h\,, \tag{29}$$

$$\{F^{\,\mathrm{pl}}\} = \sum_{e=1}^{N}\,\{F^{\,e\mathrm{pl}}\}\,, \tag{30}$$

$$\{F^{\,e\mathrm{pl}}\} = [B^e]^{\,\mathrm{t}}\,[D^e]\,\{\varepsilon^{e\mathrm{pl}}\}\,S^e h\,. \tag{31}$$

In deriving Eqs. (28)–(31) it should be kept in mind that in Eq. (16) ε should be replaced by $d\varepsilon$. This is admissible, since according to the general procedure, at each time step Δt the displacements are small.

In the case of an elastoplastic behavior of a material, the algorithm of calculations is somewhat more complex. In calculations the Mises condition is used:

$$\bar{\sigma} = \sigma_{\mathrm{y.p}}\,, \tag{32}$$

where $\bar{\sigma} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\sigma_{xy}^2}$; $\sigma_{\mathrm{y.p}}$ is the extension yield point; σ_{xx}, σ_{yy}, and σ_{xy} are the components of the stress tensor.

At the initial instant of time and as long as the moisture gradient in the material is not high, stresses in it are also low, and the intensity of the stress deviator $\bar{\sigma}$ does not exceed the yield point $\sigma_{\mathrm{y.p}}$. The body behaves as an elastic one, and the local matrix of rigidity is calculated from Eq. (20). But when stresses in the material increase to such an extent that the intensity of the stresses of deviator $\bar{\sigma}$ on the element becomes higher than the yield point $\sigma_{\mathrm{y.p}}$, the element is to be considered plastic, and in this case calculation of the rigidity matrix is performed from Eq. (29). Then the rigidity matrix of the entire material $[K]$ is formed on the basis of the matrices $[K^e]$ and $[K^{e\mathrm{pl}}]$. In the computational program we pass to the plastic state of the element if $\bar{\sigma}$ is in the range of values $\sigma_{\mathrm{y.p}} \pm 0.01\sigma_{\mathrm{y.p}}$. The magnitude of the increments of stresses in the material was controlled by a step in time dt. The step was selected so that the increment $\Delta\bar{\sigma}$ cannot exceed $0.02\sigma_{\mathrm{y.p}}$. When $\bar{\sigma}$ occurs in the range indicated, we make a correction of the components of elastic stresses. For this purpose, we calculate the value of the ratio $r = \sigma_{\mathrm{y.p}}/\bar{\sigma}$ and then multiply it by the components σ_{xx}, σ_{yy}, and σ_{xy}. Based on the products obtained we calculate the matrix $[D^{\mathrm{pl}}]$ for plastic elements. We also note that at the stage of loading, i.e., when stresses in the material increase, the term $\{F^{\,\mathrm{pl}}\}$ in Eq. (28) is not used.

As the element had passed into the plastic region, elastic deformations were fixed, and the plastic deformations were calculated from the formula

$$\varepsilon_{ij}^{\mathrm{pl}} = \varepsilon_{ij}^{\mathrm{tot}} - \varepsilon_{ij}^{\mathrm{el}} \tag{33}$$

(where $\varepsilon_{ij}^{\mathrm{tot}}$, $\varepsilon_{ij}^{\mathrm{el}}$, and $\varepsilon_{ij}^{\mathrm{pl}}$ are the total, elastic, and plastic deformations of the element) until there occurred unloading on the element and it returned to the elastic region. As the condition of transition from the plastic region to the elastic one, we adopt the state in which the entire deformation $\varepsilon_{ij}^{\mathrm{tot}}$ obtainable by calculation becomes equal to the elastic deformation $\varepsilon_{ij}^{\mathrm{el}}$ formed at the yield point. In this case, plastic deformations were preserved on the element, as a result of which the elements that were in a plastic state had a larger dried area at the end of drying than elements which remained in another region all the time. At this stage (the stage of unloading) the term $\{F^{\mathrm{pl}}\}$ was taken into account in Eq. (28).

In the case of an elastic material capable of destruction the algorithm of calculation is somewhat different; it was suggested by the author in [40]. For a two-dimensional plane stressed state the Mises rigidity condition has the form

$$\bar{\sigma} = \sigma_{\mathrm{u.s}} \,, \tag{34}$$

where $\sigma_{\mathrm{u.s}}$ is the extension ultimate strength. The difference of this procedure from the procedure for an elastoplastic material is that the unloading stage drops out, since the elements that underwent destruction are not recovered. Therefore, for an elastic body it is sufficient to use the system of equations (17).

The main stages of computation proceed as follows. At the initial instant of time and as long as the moisture gradient in the material is not high, the stresses in it are also low, and the intensity of the stress deviator $\bar{\sigma}$ does not exceed the ultimate strength $\sigma_{\mathrm{u.s}}$. The body behaves as an elastic one, and the rigidity matrix is calculated from Eq. (20). But when the stresses in the material increase so that the intensity of stress deviator $\bar{\sigma}$ on the element exceeds the ultimate strength $\sigma_{\mathrm{u.s}}$, the element can be considered destroyed, and in this case calculations of the rigidity matrix is made from Eq. (29), where in calculating $[D^{\mathrm{pl}}]$ we assume that $\varphi = 0.001$. In the program we pass to the destruction of the element when $\bar{\sigma}$ is in the range $\sigma_{\mathrm{u.s}} \pm 0.01\sigma_{\mathrm{u.s}}$. The value of the increment of stresses in the material was controlled by the step in time dt. It was selected so that $\Delta\bar{\sigma}$ could not exceed $0.02\sigma_{\mathrm{u.s}}$.

The method of calculation of mass transfer from Eq. (2) was borrowed from [31, 32]. In conclusion we note that both $[B^e]$ and S^e are calculated in terms of the coordinates of the nodes of the element; consequently, neither $[K]$ nor $\{F\}$ remains constant and they are determined at each time step. The recalculations of the rigidity matrix and column-vector of loading is made also in determining the humidity field.

The procedure suggested makes it possible to study large displacements and deformations of a body during its drying using simple rheological properties of disperse materials; it can be used in developing the techniques of their calculation for bodies with a more complex rheology and heat- and mass transfer processes.

Discussion of Results. Two cases were studied: deformation of a material in the process of drying with a free lower boundary and with a limited possibility of motion of the lower boundary. In its initial state the material has the moisture content W_0 = 1 kg/kg; an equilibrium moisture content W_{eq} = 0 kg/kg is prescribed on the surface. The coefficient of moisture conductivity was adopted equal to $a_{\mathrm{w}} = 3 \cdot 10^{-9}$ m^2/sec, the mass transfer coefficient to $\alpha_{\mathrm{w}} = 3 \cdot 10^{-6}$ m/sec, the elasticity modulus to $E = 3 \cdot 10^7$ Pa, the Poisson coefficient to $\nu = 0.5$, and the coefficient of linear shrinkage of the material to $\beta = 0.5$.

In [30], visual information on the behavior of an elastic body in drying is given. As the material is being dried, the gradient of moisture content appears in it which grows in the direction from the upper boundary to the lower one. This growth is accompanied by increasing inadmissible shrinkage of the body and, consequently, of the stress. The material alters its shape because of the curtailment of the dimensions due to the decrease in the moisture content and to the stresses appearing in it. A maximum change in the shape occurs with a maximum formation of moisture gradient over the height of a sample. In the process of drying, when the gradient of moisture content in the material becomes smaller, the stresses decrease, tending to zero at the end of drying, and the body regains the initial shape but of smaller dimensions, corresponding to the new value of moisture content. Moreover, no residual deformations and stresses remain at the end of drying.

Maximum values of stresses σ_{xx} appear on the upper and lower boundaries, and they decrease as the central plane of the body parallel to the axis x is approached. But if we fix a certain plane parallel to this axis, then the

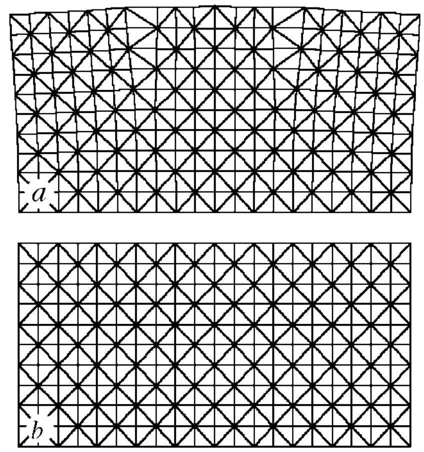

Fig. 1. States of materials at the end of drying: elastoplastic (a) and elastic (b) rheological bodies.

maximum value of σ_{xx} is attained at the center of the material, and it drops to zero on the side boundaries of the body. The quantity σ_{yy} has a maximum at the center of the side boundary planes and decreases at their edges. The component σ_{xy} is equal to zero on the central and side surfaces of the material, taking maximum values in the plane between the central portion of the material and its side surfaces.

Calculation shows that in a body with a moving lower boundary, stresses smaller in absolute magnitude are developed than those in the sample in which the lower boundary cannot move in the vertical direction. This is explained by the fact that free deformation partially relieves the unaccounted shrinkage caused by the moisture gradient.

The study of a material with elastoviscous rheology (additionally it was adopted that the relaxation time $t_{\rm r}$ = 3600 sec and the Poisson coefficient $\nu = 0.35$) shows that its behavior resembles deformation of a purely elastic body (see above). We will consider in more detail the characteristic features of elastoviscous deformation. The stresses formed in an elastoviscous material are smaller than those formed in a purely elastic one. This is due to the presence of viscous properties of the medium, which leads to a greater mobility of its structure and, consequently, to the removal of a portion of stresses in it. Visual observation of the process of deformation showed that at the end of drying the body that initially had a rectangular shape regains this shape, and at the end of drying the stresses disappear from it, i.e., no residual deformations and stresses are formed. This is explained by the fact that the process of drying occurs in two stages. In the first stage, on increase in the moisture content gradient the material is loaded and viscous and elastic deformations are formed in it, and in the second stage, when the moisture content gradient decreases, the material is unloaded and simultaneously both elastic and viscous deformations decrease, and they disappear by the end of drying. We note that elastic deformations disappear as a result of the drop in the moisture content gradient, whereas the viscous ones — due to the process of relaxation that closely follows the decrease in the moisture content gradient. Whence the conclusion can be drawn that on completion of drying the elastoviscous deformations do not lead to the appearance of residual deformations, even temporal ones.

Based on the procedure developed, we calculated the process of drying of an isotropic elastoviscous material with the characteristics given above, as well as with the Poisson coefficient $\nu = 0.5$ and the value for the yield point $\sigma_{\rm y.p} = 3\cdot10^6$ Pa. We considered a material with a limited possibility of motion for its lower boundary in the process of drying.

The maximum extending stresses in drying appear on the upper side of the material, whereas maximal compressing ones are realized on the lower side. On increase in the yield point, some of the elements pass into a plastic state and thus fix stresses on the upper side, preventing their increase. The stresses, however, continue to increase on the underlying layers. If the yield point is also exceeded on them, a portion of the elements will pass into a plastic state. The process will continue until the moisture content gradient in the material begins to decrease followed by a decrease in stresses, and all the elements again pass into the elastic region. Plastic deformations lead to a situation where the elements subjected to them will shrink less than the elements that did not undergo such deformations. As

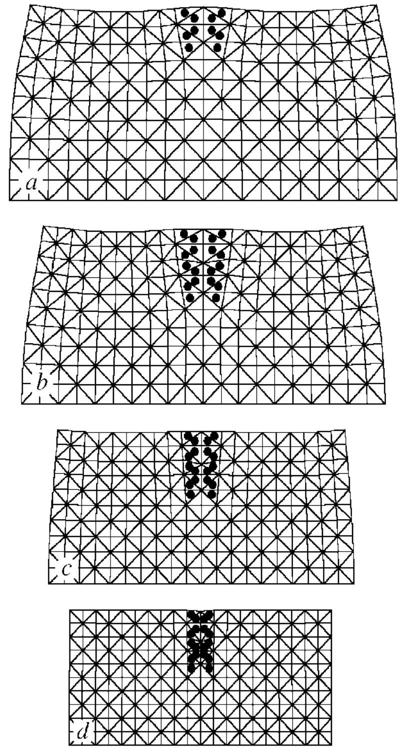

Fig. 2. Material with cracks at different instants of time t: a) 9020; b) 38,620; c) 106,620; and d) 301,020 sec.

the body is dried up further, the extending stresses in the upper layers will decrease, passing through zero value, and then they will go over into compressing ones. In the lower layers the reverse picture is observed, as a result of which they are subjected to extending stresses. As a result, the upper layers will be compressed by the lower ones and the latter will be extended by the upper ones. On complete drying of the material the residual stresses will take maximum values. Figure 1 demonstrates the states of the materials at the end of the process of drying with elastoplastic and elastic properties, respectively. It is seen from this figure that the elastoplastic material, in contrast to elastic one (and elastoviscous one), failed to regain the initial shape. The results obtained agree with the data given in [41, 42].

Based on the procedure developed, calculations of the process of drying of an isotropic elastic disperse medium with the possibility of its destruction have been made. A material with a limited possibility of motion of the lower boundary in the process of drying for the ultimate strength $\sigma_{u.s} = 4 \cdot 10^6$ Pa was considered. Figure 2 presents an example of calculation at different time instants. The points denote the elements at which a break in continuity occurrs. The discontinuity begins on the upper boundary and propagates inward. In this case the crack becomes wider, which leads to the redistribution of the load in the material. As dehydration proceeds and the moisture content gradient falls the material regains its initial shape and the crack closes up.

Conclusions. It is shown that for both more accurate calculations and development of the theory of drying the system of interrelated equations of heat and mass transfer must also include equations of mechanical motion. Basic processes responsible for the drying of materials liable to deformation are singled out. A procedure of numerical calculation of the drying of bodies with large displacements of its elements and deformations has been developed. Based on the proposed procedure, the processes of drying for the basic rheological models, elastic, elastoviscous, and elastoplastic, have been calculated. The procedure developed allows one also initiate the process of destruction of elastic materials.

NOTATION

a_w, moisture conduction (diffusion) coefficient, m^2/sec; B, bulk compression modulus, Pa; $[B]$, matrix of gradients; $[D]$, matrix of the characteristics of a material; E, modulus of elasticity, Pa; e_{ij}, deviator of deformations; $\{F\}$, global vector-column of loading; G, shear modulus, Pa; G_0 and G_∞, instantaneous and extremely long shear moduli, Pa; h, thickness of material, m; $[K]$, global stiffness matrix; $[K^e]$, $[K^{e\text{pl}}]$, local stiffness matrices; M, number of steps in time; N, number of elements; $[N]$, matrix of shape functions; $R_s(t)$ and $R_v(t)$, functions of the rates of shear and bulk relaxations; $\{R\}$, $\{r\}$, vectors taking into account the hereditary properties of the material; S, area, m^2; s_{ij}, stress deviator, Pa; T, temperature, K; t, time, sec; Δt, step in time, sec; t_r, time of relaxation, sec; $\{U\}$, vector of nodal displacements; $\{u\}$, vector of displacements; V, volume, m^3; W, moisture content, kg/kg; W_{eq}, equilibrium moisture content; kg/kg; x, y, coordinates, m; α_w, mass transfer coefficient, m/sec; β_v, coefficient of volume shrinkage; γ, shear deformation; δ, symbol of mathematical operation of variation; δ_{ij}, Kronecker symbol; $\{\varepsilon\}$, strain vector; ε_{ij}, strain tensor; ν, Poisson coefficient; σ_{ij}, stress tensor, Pa; $\bar{\sigma}$, intensity of stress deviator, Pa; $\{\sigma\}$, stress vector; τ, integration variable, sec. Subscripts and superscripts: 0, initial; v, volumetric (bulk); y.p, extension yield point; e, number of element; t, transposed; eq, equilibrium; w, moisture (water); tot, total deformation; el, elastic; pl, plastic; u.s, extension ultimate strength; s, shearing; i, j, k, tensor components and running values of terms being summed up; r, relaxation.

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